Vacuum Fluid Dynamics: Electric and Gravitational Forces Linked to the Primary and Secondary Bjerknes Forces in Vacuum

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(Dated: 8/12/2011)

This paper describes theoretical work done in an attempt to find a physical mechanism for the gravitational and electric forces. This work demonstrates how both forces can be described as acoustic radiation forces (Bjerknes forces) which exist in a vacuum with fluid properties. Vacuum parameters are derived for use in both Bjerknes force equations to demonstrate the equivalency of these equations to Newton's gravitational force law and Coulomb's law. In particular, this paper will demonstrate the following equivalencies:

$$F_g = F_{B2} \cdot 2\pi \cdot n_1 n_2$$
$$F_e = F_{B1} \cdot \frac{\alpha}{2\pi} \cdot n_1 n_2$$

I. OVERVIEW OF THE BJERKNES FORCES

The Bjerknes forces are radiation forces exerted by an acoustic field on gas bubbles in a fluid. They are a highly important set of forces in the fields of sonoluminescence, cavitation, acoustic degassing, and medical ultrasonics [1]. There are two Bjerknes forces, called primary and secondary. The primary force is due to an external sound field, and results in the attraction or repulsion of single bubbles at a pressure node or antinode. The secondary force is due to the sound fields emitted by other bubbles and also results in mutual attraction or repulsion [2]. The forces are named after C. A. Bjerknes and his son V. F. K. Bjerknes, the first to describe their effects [1].

This paper makes a direct link between the electric force and the primary Bjerknes force, and with the gravitational force and the secondary Bjerknes force.

II. DESCRIPTION OF FINDINGS

A. The Gravitational Force

The different parts of this section describe calculations that were done to arrive at two necessary parameters for the secondary Bjerknes force equation: pressure amplitude A (from radiation pressure P_{rad}), and density ρ. The line of reasoning leading up to the values of A and ρ is given below.

1. *Using the distributed inductance and capacitance of free space to find radiation pressure*

Space is known to carry distributed inductance and capacitance by virtue of the values of free space permittivity and permeability. First we begin by setting the energy of a photon equivalent to the electron's rest mass energy to be equal to the energy stored in either a free space inductor or capacitor.

$$U_c = hf_c = \frac{1}{2}CV^2 \tag{1}$$

$$U_L = hf_c = \frac{1}{2}LI^2 \tag{2}$$

Next we solve for voltage V and current I, selecting r_c as our unit of distance over which we will calculate the inductance and capacitance. This means that $C = \epsilon_o r_c$ and $L = \mu_o r_c$. Substituting these into the above equations yields

$$V = \sqrt{\frac{2hf_c}{\epsilon_o r_c}} \tag{3}$$

$$I = \sqrt{\frac{2hf_c}{\mu_o r_c}} \tag{4}$$

Now we find the values of the electric and magnetic fields E and B using the formulas for energy density of the E and B fields. Note that these equations work when $r = r_c$ (Compton wavelength).

$$\frac{U_c}{r^3} = \frac{1}{2}\epsilon_o E^2 = \frac{1}{2}\epsilon_o \frac{V^2}{r_c^2} \tag{5}$$

$$\frac{U_L}{r^3} = \frac{1}{2}\frac{B^2}{\mu_o} = \frac{1}{2}\mu_o \frac{I^2}{r_c^2} \tag{6}$$

Solving for E and B,

$$E = \frac{\sqrt{\frac{2hf_c}{\epsilon_o r_c}}}{r_c} \tag{7}$$

$$B = \frac{\mu_o}{r_c}\sqrt{\frac{2hf_c}{\mu_o r_c}} \tag{8}$$

Substituting both these equations into the equation for the Poynting vector $S = \frac{1}{\mu_o}EB$ allows the determination of a value for radiation pressure P_{rad} using the equation $P_{rad} = \frac{S}{c}$. This value is

$$P_{rad} = 1.1464 \times 10^{22} Pa \tag{9}$$

2. Finding values for vacuum density and bulk modulus

These values are determined from dimensional analysis, and again assuming the common distance scale r_c which is equal to the Compton wavelength. The procedure for figuring vacuum density ρ is below; mass is written in terms of charge according to the classical electron radius formula, where any mass m_o can be imagined to have an electromagnetic radius equal to r.

$$\rho \rightarrow \frac{kg}{m^3} = \frac{m}{r^3} = \frac{\mu_o q^2}{4\pi r} \cdot \frac{1}{r^3} = \frac{\mu_o q^2}{4\pi r^4} \tag{10}$$

For vacuum density, the Planck charge q_{pl} is used for q, and the Compton wavelength r_c is used for r. Other combinations of parameters have been tested, and only these work for the secondary Bjerknes force equation. These parameters lead to the following value for vacuum density:

$$\rho = \frac{\mu_o q_{pl}^2}{4\pi r_c^4} = 1.0150 \times 10^4 \frac{kg}{m^3} \tag{11}$$

The same procedure can be done for the vacuum bulk modulus B, which has units of $\frac{N}{m^2}$.

$$B \rightarrow \frac{N}{m^2} = \frac{kg}{ms^2} = \frac{m}{rt^2} = \frac{\mu_o q^2}{4\pi r^2 t^2} \tag{12}$$

The following expression for B is obtained after substituting $c^2 = \frac{1}{\epsilon_o \mu_o}$ in the above calculation:

$$B = \frac{q^2}{4\pi\epsilon_o r^4} = \frac{q_{pl}^2}{4\pi\epsilon_o r_c^4} = 9.1224 \times 10^{20} Pa \tag{13}$$

Comparing the above result to Eq. 9, the following relationship can be noted:

$$B = \frac{P_{rad}}{4\pi} \tag{14}$$

This value of B is what we will use in the secondary Bjerknes force equation as the value of the pressure amplitude A.

3. F_g as the secondary Bjerknes force in vacuum

The gravitational force on a mass m_1 by a mass m_2 is given by the following equation, Newton's gravitational force law:

$$F_g = G\frac{m_1 m_2}{r^2} \tag{15}$$

This section will show that

$$F_g = F_{B2} \cdot 2\pi \cdot n_1 n_2 \tag{16}$$

where $n_1 = m_1/m_e$ and $n_2 = m_2/m_e$. The secondary Bjerknes force equation is as follows:

$$F_{B2} = \frac{2\pi |A|^2 \omega^2 R_{10} R_{20}}{\rho L^2 (\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2)} \tag{17}$$

where A = acoustic pressure amplitude, ω = the acoustic driving frequency, R_{10} and R_{20} are the equilibrium radii of the two bubbles in question, ρ is the fluid density, L is the distance between bubbles, and ω_1 and ω_2 are the resonance frequencies of the two bubbles, respectively [3].

Eq. 16 can be validated as follows. By using the following values in Table I, the secondary Bjerknes force equation can be used in conjunction with Eq. 16 to calculate the gravitational force by the Earth on a 1 kg mass on the Earth's surface. This value should be compared to that obtained from Newton's gravitational force law for the values of m_1, m_2, and L in Table I.

The values of n_1 and n_2 scale the secondary Bjerknes force equation up from the scale of the electron's Compton wavelength (single bubbles) in order to calculate the gravitational force at macroscopic scales (large bubble clusters).

B. The Electric Force

The electric force on a test charge q_1 by a point charge q_2 is given by the following equation:

$$F_e = \frac{q_1 q_2}{4\pi\epsilon_o r^2} = qE \tag{18}$$

The electric force is much stronger in comparison to the gravitational force, as is typically the case for the primary Bjerknes force compared with the secondary Bjerknes force. This section will demonstrate the relationship between the primary Bjerknes force and the electric force, which is mediated by the fine structure constant α :

$$F_e = F_{B1} \cdot \frac{\alpha}{2\pi} \cdot n_1 n_2 \tag{19}$$

where $n_1 = q_1/q_e$ and $n_2 = q_2/q_e$. The equation for the primary Bjerknes force is

$$F_{B1} = -V\nabla P \tag{20}$$

where V = the time-averaged volume of the bubble and ∇P = the time-averaged acoustic pressure gradient. Note that this is in the form of $F = qE$; charge can be thought of a kind of oscillating bubble volume, while E can be thought of as a kind of oscillating pressure gradient in the vacuum fluid.

Given that ∇P is a pressure gradient, its value must reflect a pressure over a distance. The vacuum bulk modulus $B = \frac{P_{rad}}{4\pi}$ shall be used as the pressure, and the Compton wavelength of the electron r_c shall be used as the distance across which the pressure gradient occurs. Combining Eqs. 19 and 20 gives the following:

$$F_e = -V\nabla P \cdot \frac{\alpha}{2\pi} = -V \cdot \frac{\frac{B}{4\pi}}{r_c} \cdot \frac{\alpha}{2\pi} \tag{21}$$

The final parameter to be addressed is volume. Equation 19 holds only for a volume equal to $2\pi r_c^3$, which

Parameter	Description	Selected Value	Numerical Value
A	acoustic pressure amplitude	vacuum bulk modulus B	$9.1224 \times 10^{20} Pa$
ω	acoustic driving frequency	$2\pi f_{pl}$ where f_{pl} = the Compton frequency of a Planck mass	$1.8549 \times 10^{43} Hz$
R_{10} and R_{20}	equilibrium radii of the two bubbles	the electron Compton wavelength, r_c	$2.4263 \times 10^{-12} m$
ρ	fluid density	vacuum density ρ	$1.0150 \times 10^4 \frac{kg}{m^3}$
L	distance between bubbles	mean earth radius	$6.3675 \times 10^6 m$
ω_1 and ω_2	resonance frequencies of the two bubbles	$2\pi f_c$ where f_c = the electron Compton frequency	$7.7634 \times 10^{20} Hz$
n_1	how much greater mass m_1 is than the electron mass m_e	$n_1 = m_1/m_e$ where $m_1 = 1kg$ and $m_e = 9.1094 \times 10^{-31} kg$	1.0978×10^{30}
n_2	how much greater mass m_2 is than m_e	$n_2 = m_2/m_e$ where $m_2 = 5.9739 \times 10^{24} kg$	6.5580×10^{54}

TABLE I. Table of values to use in the secondary Bjerknes force equation to test the equivalence of Eq. 16 with Newton's gravitational force law.

Parameter	Description	Selected Value	Numerical Value
V	bubble volume	volume of a cylinder or spherical torus equal to $2\pi r_c^3$	$8.9747 \times 10^{-35} m^3$
B	pressure amplitude of the pressure gradient	vacuum bulk modulus B	$9.1224 \times 10^{20} Pa$
r_c	distance over which the pressure gradient occurs	the electron Compton wavelength	$2.4263 \times 10^{-12} m$
α	fine structure constant	fine structure constant	7.2974×10^{-3}
q_e	electric charge	elementary charge of an electron	$1.60217646 \times 10^{-19} C$
ϵo	permittivity of free space	permittivity of free space	$8.85418782 \times 10^{-12} m^{-3} kg^{-1} s^4 A^2$

TABLE II. Table of values to use in Eq. 22 to test the equivalence of the primary Bjerknes force expression with Coulomb's law.

interestingly is not the volume of a spherical bubble but of a cylinder or spindle torus.

To show the equivalency between Eqs. 18 and 19, the numbers in Table II should be used in Eq. 22 below.

$$F_e = -V \cdot \frac{\frac{B}{4\pi}}{r_c} \cdot \frac{\alpha}{2\pi} = \frac{q_1 q_2}{4\pi\epsilon_o r^2} = \frac{q_e^2}{4\pi\epsilon_o r_c^2} \tag{22}$$

III. DISCUSSION

What is matter, and where do the gravitational and electric forces come from? This work suggests a picture of matter as composed of oscillating bubbles in a universal fluid, acted upon by an external acoustic field. It is a simple, parsimonious model which explains both the gravitational and electric forces under a common mechanism. Thus, further investigation into the relationship between these fundamental forces and Bjerknes forces in a vacuum fluid is encouraged.

A. The Bjerknes Forces

1. *Primary Bjerknes Force*

The primary Bjerknes force is central to studies of sonoluminescence, whereby an air bubble inside a flask insonified by a strong external sound field is made to suddenly collapse, emitting light. The bubble is trapped at the center of the flask by the primary Bjerknes force, and this trapping is a combined effect of the sound field and nonlinear bubble oscillations [4].

A body of time-averaged volume V in a liquid under a time-averaged acoustic pressure gradient ∇P experiences a force

$$F_{B1} = -V\nabla P \tag{23}$$

[4][5]. For a spherical bubble, the volume would be that of a sphere of a particular equilibrium radius, but the above equation is not limited to spherical bodies.

The physical mechanism of the Bjerknes force is illustrated in greater detail in [5], but a brief description is as follows. This description is for the case of small driving pressures and drive frequencies below the bubble's natural resonance frequency. The primary Bjerknes force arises from a pressure gradient across the bubble - a slight difference in pressure exerted on either side of the bubble's surface. During the tensile phase of the sound field, the pressure force directs the bubble toward the pressure antinode. During the compressive phase, the pressure force directs the bubble away from the pressure antinode. However, because the bubble is smaller during the compressive phase than the tensile phase, the pressure force is smaller during the compressive phase, so the time-averaged force is in the same direction as that of the tensile phase–toward the pressure antinode.

In the case of bubbles driven above their natural resonance frequency, as is the case here, a different phase response occurs that drives bubbles away from the pressure antinode and toward a node [5].

2. Secondary Bjerknes Force

The secondary Bjerknes force results in the mutual interaction between oscillating gas bubbles in a sound field:

$$F_{B2} = \frac{2\pi |A|^2 \omega^2 R_{10} R_{20}}{\rho L^2 (\omega_1^2 - \omega^2)(\omega_2^2 - \omega^2)} \tag{24}$$

where A = acoustic pressure amplitude, ω = the acoustic driving frequency, R_{10} and R_{20} are the equilibrium radii of the two bubbles in question, ρ is the fluid density, L is the distance between bubbles, and ω_1 and ω_2 are the resonance frequencies of the two bubbles, respectively [3].

Weakly driven bubbles of a fixed equilibrium radius R_{10} or R_{20} show a maximum response at their linear resonance frequency ω_1 or ω_2. If the driving frequency lies between ω_1 and ω_2 the bubbles will repel each other; otherwise the force is attractive [2].

B. Future Directions

As to the question of whether these fundamental forces may be alterable, this work suggests the possibility. This explanation of the gravitational and electric forces makes the assumption of an incompressible vacuum fluid. If compressibility is a factor, this can introduce nonlinearities in the oscillations that give rise to new behaviors [2]. In one case, the sign of the secondary Bjerknes force can be reversed as two bubbles approach each other [6].

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