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Memories in sand: Experimental tests of construction history on stress distributions under sandpiles

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We report experiments on piles of cohesionless granular materials showing the effect of construction history on static stress distributions. Stresses under piles are monitored by sensitive capacitive techniques. The piles are formed either by pouring granular material from a funnel with a small outlet (localized source), or from a large sieve (homogeneous rain). Localized sources yield stress profiles with a clear stress dip near the center of the pile; the homogeneous rain profiles have no stress dip. We show that the stress profiles scale linearly with the pile height. Experiments on wedge-shaped piles show similar but weaker effects.
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Granular systems have captured much recent interest because of their rich phenomenology, and important applications [1]. Static arrays show inhomogeneous spatial stress profiles called stress chains [2], where forces are carried primarily by a small fraction of the total number of grains (see Fig. 1). Recent numerical simulations [3] and experiments [4] have shown that the structure and the nature of these chains plays a critical role in the dynamics and statics of dense granular systems even in the absence of strong disorder of the granular packings [5,6]. Necessarily, the presence of these chains must be reflected in the continuum constitutive relations needed to close the governing equations and to solve even the simplest static boundary value problems [7–11]. The stress profile under a static pile of granular material provides a useful method for probing the effects of stress chains and the history of their formation. The literature contains many experiments [9,11] and simulations [12] examining stress profiles under static piles of granular material. Although there are a number of such studies, they are not in mutual agreement, and competing constitutive models have been invoked to explain the experiments. Of possible pile geometries, conical and wedge-shaped heaps have been the most frequently studied. Many of the experiments on conical piles have indicated, contrary to simple intuition, that there is a dip in the pressure profile beneath the center [13–16]. A stress dip is also reported in recent soft-particle simulations [12]. The existence of a dip in the stress profile for wedge-shaped piles is an open question [9,17], and we are aware of only one set of experiments [13,19] for this case that explores construction history [19]. These authors formed piles by three methods. The different techniques yielded results that were identical within the resolution of their instruments; no dip was recorded.

The present experiments were done to address experimental conflicts and to test theories that depend explicitly on construction history [8,11] by determining as carefully as possible the relation between the preparation of a heap and the stress profile at its base. We explore the effects of construction procedures on the pressure profiles using two different methods to build both conical and wedge-shaped heaps.

There are important technical considerations in determining whether there is a stress dip. The most important of these is that even modest deformations of the surface supporting the pile or the force detector may lead to erroneous measurements [9]. Also, if the pile is formed by dropping material onto the heap from a considerable height, versus gentler methods, it is likely that kinetically induced stresses become frozen into the heap. In this case, or for a heavy load, there may be a characteristic length associated with the deformation of the pile under its own weight.

Several details of the present experiments are important. We used sand of diameter 1.2 mm±0.4 mm and angle of repose 33°. The base plate on which we constructed most of the piles was 15.0 mm thick duralumin, which was adequate to prevent deflection under the weight of the pile. (Some additional experiments were made using a 1.3 cm steel base and a fixed funnel height.) For a typical sand pile of final height $H=8$ cm, we estimate the maximal sagging of the bottom plate to be $w_m=6.5$ μm. Therefore $w_m/H=10^{-5}$, a value that was smaller by $\sim 10^{-3}$ than the relative deflection for which sagging of the base might create a significant perturbation [17,19,20]. A single capacitive normal stress (i.e., pressure) sensor of diameter of 11.3 mm (9 grain diameters) was placed flush with the surface of the base plate. We then determined the normal stress at various locations along the radial axis of the conical piles or along the short edge of the wedge-shaped piles by repeated construction of heaps with

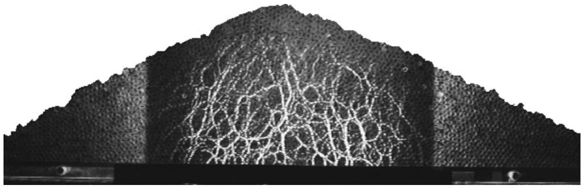

FIG. 1. Two-dimensional pile of photoelastic disks (diameters 0.74 and 0.9 cm) created by a localized-source procedure. The center section of the image, with a height of ∼30 cm, is viewed between crossed polarizers, allowing one to see the underlying stress structure.

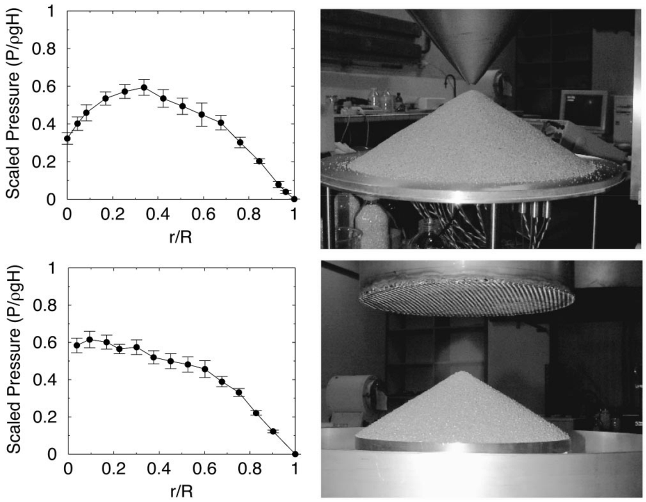

FIG. 2. Dimensionless normal stress profiles, $P/\rho gH$, vs dimensionless radial distance r/R, beneath conical piles of granular materials of height H and radius R. The construction techniques are illustrated by the accompanying photographs (see text).

the same mass of sand. The resolution of the measuring device [18] was 0.25% of the typical maximum stress for an 8 cm pile, corresponding to a vertical deflection of the sensor of ~ 1.3 μm. We made measurements with different membrane thicknesses, and we found consistent results within experimental resolution. Here, we present data obtained with only one of these membranes, which had a thickness $t=100$ μm. The sensor was calibrated against the hydrostatic pressure of a water column. However, the response of the sensor to known weights of granular material was consistently somewhat smaller, by a factor of ~ 0.9, than for water. We emphasize that this reduction was constant throughout the measurements. In particular, using a calibration based on granular mass, we generally found that the integrated weight of the pile was correct.

We constructed both types of heaps by two qualitatively different procedures. The first, a ''localized source'' procedure used a funnel; the second, a ''raining procedure'' used a sieve. In the following paragraphs, Figs. 2, and 3 give details and photographs.

The localized-source procedure: We formed the pile using a funnel with an outlet that was much smaller than the final pile diameter. The funnel lifted steadily, with the outlet always slightly above the apex. This approach, versus a fixed funnel height, avoided the deposition of particles with large kinetic energies that varied with the heap height [13,14]. For conical piles, the sand emptied from a conical funnel with outlet diameter 11.7 mm ($\simeq 10$ grains) onto the duralumin plate, forming a heap of base diameter $2R$, smaller than the plate. For wedge-shaped piles the sand emptied from a wedge-shaped funnel with an outlet that was 1.17 by 20 cm The final heap completely covered the base (dimensions 20 $\times 26$ cm). Two Plexiglas walls 2.0 cm thick and taller than the pile bounded the heap on two sides; the two sides parallel to the long direction of the wedge were open. The sensor was placed halfway between the walls and at various distances from the centerline of the heap. During the experiments we measured the volume of the pile, which with the known mass, yielded the average density, ρ.

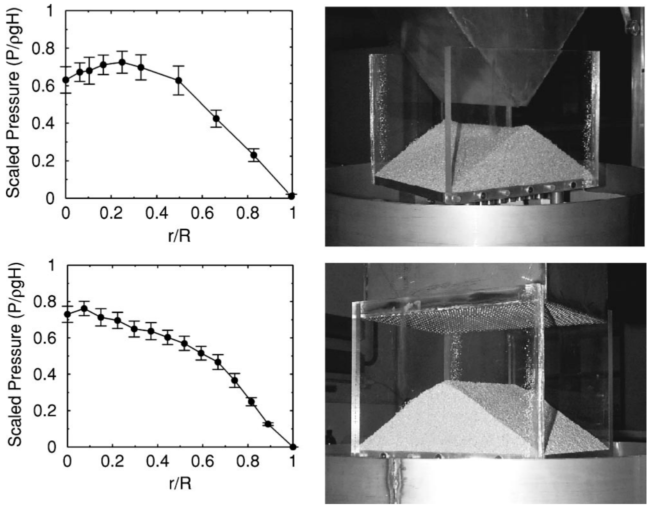

FIG. 3. Dimensionless normal stress profiles, $P/\rho gH$, vs transvserse distance r/R, beneath wedge-shaped piles of granular material of height H and width $2R$. The piles are made by different construction techniques illustrated by the accompanying photographs (see text).

The raining procedure: This method was designed to align the stress chains more nearly in the vertical direction. The sand was poured from containers with cross-sectional dimensions slightly larger than the platform, and whose bottoms were wire meshes with 0.40 cm diameter holes. The containers were filled while resting on the platform and then raised slowly, allowing a steady rain of sand onto the heap. Excess sand was allowed to avalanche off the platform. For this procedure, the final heap covered the platform. For conical piles we used a cylindrical container and a supporting platform of diameter 26 cm (236 grain diameters). For the wedge-shaped piles, we used a rectangular box with dimensions 20×26 cm; the platform was identical to the one used in the localized-source procedure. At the end of the procedure, we measured the mass volume of the pile.

Pressure profiles and photographs of the final conical and wedge-shaped piles are shown in Figs. 2. The distance from the center of the heap is scaled by R, which is the final pile radius for conical heaps, and the final distance from the center axis for wedge-shaped heaps. The pressure is scaled by the hydrostatic pressure, ρgH. The bars represent the standard deviation of several *independent* runs, typically 10 to 12, not experimental error, which is about 0.25%.

The entire weight of each pile was determined by integrating a curve fitted to the profile. These calculations and the known weights of the piles agreed to about 1.5% or less for all cases except the wedge-shaped piles generated by the raining procedure. For that case, we observed a discrepancy as large as 8%. This relatively large ''missing mass'' may be caused because the walls support some of the weight.

Data for the conical piles created by the localized-source method show a clear pressure minimum at $r/R=0$. A maximum in the stress of $\sim 0.6\rho gH$ occurs at a position $r/R \cong 0.3$, which agrees reasonably well with previous conical pile data [15,16]. Experiments performed with a fixed height funnel show a larger pressure difference between the maximum at $r/R=0$ and the value at $r/R=0.3$. This suggests that the particles pack differently with different deposition energies.

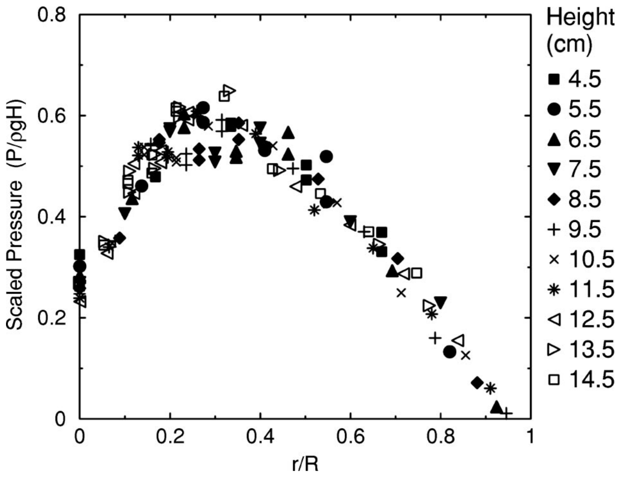

FIG. 4. Dimensionless normal stress profiles, $P/\rho gH$, vs dimensionless radial position, r/R, for different pile heights H in localized source experiments for a conical pile.

A dip does not occur in the profiles of the heaps created by the raining method. Rather, there is a peak pressure of about 0.6 at $r/R=0$, and a steady drop in the pressure moving out towards the edge of the pile.

For the wedge-shaped piles we find results that are qualitatively similar to those for conical piles. With the raining procedure, the stress profile shows no indication of a central dip. With the localized-source there is a clear minimum at $r/R=0$. The value of this dip is significantly smaller than for the analogous conical heap, i.e., only 15% lower than the maximum stress, rather than 50% lower. The pressure at the center is about $0.65\rho gH$. The maximum in the stress occurs at $r/R\cong 0.25$ with a value of about $0.75\rho gH$. While the dip is smaller than the conical pile case, there is a definite variation in the shapes of the profiles

An important question concerns the dependence of the stress profile on heap size. Earlier experiments [14,15] suggested that the size and scaled position of the stress maximum vary with the size of the pile. Alternatively, Radjai [21] has suggested that the relative sizes of the funnel opening and the heap are important.

We probed the issue of heap size by constructing conical piles with the localized source procedure for heap heights spanning $4.5\ \text{cm}\leqslant H\leqslant 14.0\ \text{cm}$. We did so by stopping the filling process at various stages to obtain stress data. This variation by ~ 3 in the maximum height of the piles corresponded to a variation of ~ 30 in the mass. The resulting data are displayed in Fig. 4. Within the scatter, the normalized profiles collapse well. The peak occurs consistently at $r/R=0.3$, and the stress at $r=0$ is consistently $\sim 50\%$ of the peak stress. This finding disagrees with earlier studies by Jotaki and Moriyama [14], involving conical piles formed by pouring from funnels. These authors found that that the larger piles had deeper dips in the stress at the center. The difference between this data and ours is that Jotaki and Moriyama used a fixed funnel height for a given heap height. Larger piles were formed by setting the funnel progressively higher. The height dependence observed by Jotaki and Moriyama may be explained by density differences in the packings induced by the variable energy of deposition. In this regard, we recall that when we fixed the funnel height at $z>H$, the stress dips were deeper than when we gradually raised the funnel.

To conclude, we have shown that the construction history affects the pressure distribution at the bottom of conical and wedge-shaped piles formed on a rigid base. We observed a pressure dip at the center of a pile if we used a localized source. The pressure profile scaled linearly with the pile height, within experimental scatter. It seems likely that the progressive formation of the pile by successive small avalanches leads to the occurrence of a pressure dip. In the case of a more uniformly vertical filling via a raining procedure, the dip disappears. A localized-source procedure with a fixed pouring height tends to produce a height-dependent stress profile (with a dip).

The dip in these experiments cannot be caused by a deformation of the base. If small deflections of the base (order 10^{-5}) were an issue, then that effect should appear in both the localized source and raining procedures, and would also prevent the collapse of the data for different heap heights.

A heuristic explanation of the mechanism producing the dip is that the flow of particles during the localized-source procedure forms stress chains oriented preferentially in the direction of the slope (c.f. Fig 1). These chains form arches that shield the center from some of the weight, thereby forming the dip. These effects agree qualitatively with the explanations of Wittmer *et al.* [8], and with recent numerical simulations [12]. We will present additional details and a more extensive comparison to theory elsewhere.

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